Geometry Applications: Triangles

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THE YEAR WAS 1944 IN NAZI OCCUPIED FRANCE.

ADOLF HITLER HAD JUST ISSUED A STARTLING ORDER:

DESTROY THE EIFFEL TOWER.

IT WOULD BE THE LAST INDIGNITY ON THE FRENCH

DURING THE GERMAN RETREAT.

FORTUNATELY HITLER'S MEN DID NOT FOLLOW HIS ORDER.

PERHAPS THEIR DECISION WAS INFLUENCED

BY HOW STURDY THE TOWER IS.

IT WAS BUILT TO LAST.

BUILT IN 1889, THE EIFFEL TOWER REMAINS

ONE OF FRANCE'S TALLEST STRUCTURES.

IT IS OVER 320 METERS,

OR ABOUT THE LENGTH OF THREE FOOTBALL FIELDS.

AS YOU TRAVEL FROM GROUND LEVEL TO THE VERY TOP

YOU WILL SEE A NUMBER OF TRIANGULAR SUPPORTS.

GUSTAVE EIFFEL, THE DESIGNER OF THE TOWER,

WAS ORIGINALLY A BRIDGE BUILDER BEFORE HE BECAME

THE CREATOR OF THE TOWER THAT BEARS HIS NAME.

MANY OF THE BRIDGES THAT EIFFEL BUILT

RELIED ON A STRUCTURAL SUPPORT CALLED A TRUSS.

A TRUSS IS A TRIANGULAR SHAPED SUPPORT

THAT ALLOWS A BRIDGE TO SUPPORT HEAVY WEIGHT.

HERE'S HOW A TRUSS WORKS.

IMAGINE TWO PARALLEL PLANES THAT REPRESENT

THE TWO HORIZONTAL SUPPORTS OF A BRIDGE.

THE TOP PLANE IS THE PLATFORM,

WHERE A PERSON OR VEHICLE STAND, AND THE LOWER PLANE

HELPS SUPPORT THE WEIGHT OF THE PERSON.

THE TWO PLANES ARE CONNECTED BY A TRIANGULAR TRUSS.

THE POINT WHERE THE TRIANGLE MEETS THE

UPPER PLANE IS CALLED THE VERTEX OF THE TRIANGLE.

SUPPOSE THAT SOMEONE IS WALKING ACROSS THE BRIDGE

AND SETS FOOT ABOVE THE VERTEX.

THERE IS A DOWNWARD FORCE INDICATED BY THE ARROW.

THIS FORCE IS THEN REDIRECTED INTO

TWO SMALLER FORCES BECAUSE OF THE TRUSS.

THE TWO SIDES OF THE TRIANGLE EACH HAVE A

DOWNWARD SLANTING FORCE AND AS A RESULT

THE VERTICAL FORCE IS SPLIT AND REDIRECTED.

THIS IS ONE WAY THAT A TRUSS MAKES IT POSSIBLE

FOR A BRIDGE TO CARRY A LOT OF WEIGHT

BY REDISTRIBUTING IT.

LET'S TAKE A CLOSER LOOK AT THIS PHENOMENON.

CIVIL ENGINEERS REFER TO THE DOWNWARD FORCES ON THE

TWO SIDES OF THE TRIANGLE AS FORCES OF COMPRESSION.

THIS IS BECAUSE THE WEIGHT, ALSO REFERRED TO

AS THE LOAD, PRESSES DOWN ON THESE PARTS OF THE TRUSS.

THE BASE OF THE TRIANGLE EXPERIENCES

A DIFFERENT TYPE OF FORCE.

THE TWO UPPER SIDES OF THE TRIANGLE ARE POINTED

IN TWO DIFFERENT OPPOSITE DIRECTIONS.

THIS CREATES TWO OUTWARDLY POINTING FORCES.

SO THE BASE OF THE TRIANGLE EXPERIENCES

WHAT ENGINEERS REFER TO AS TENSION.

THE BASE IS PULLED IN TWO DIRECTIONS.

WITH ALL OF THESE FORCES AT WORK

WHY DOESN'T THE TRUSS FALL APART?

WITH A WELL-DESIGNED BRIDGE, ALL OF THE FORCES ARE

BALANCED IN WHAT IS CALLED STATIC EQUILIBRIUM.

BUT THIS EQUILIBRIUM IS BASED ON THE STABILITY

BROUGHT ABOUT BY A TRIANGLE.

LET'S EXPLORE THIS STABILITY ON THE TI-NSPIRE.

TURN ON THE TI-NSPIRE.

CREATE A NEW DOCUMENT.

YOU MAY NEED TO SAVE A PREVIOUS DOCUMENT.

CREATE A NEW "GRAPHS AND GEOMETRY" WINDOW.

CREATE A TRIANGLE.

PRESS MENU, AND UNDER "POINTS AND LINES"

SELECT SEGMENT.

USE THE NAV PAD TO MOVE THE POINTER

TO THE MIDDLE PART OF THE SCREEN.

PRESS ENTER TO DEFINE THE BASE OF THE TRIANGLE.

MOVE THE POINTER TO THE RIGHT

AND YOU'LL SEE THE BASE TAKING SHAPE.

PRESS ENTER AGAIN TO DEFINE THE ENDPOINT OF THE BASE.

WE WILL BE CONSTRUCTING A SPECIAL TYPE OF TRIANGLE

COMMONLY USED WITH TRUSSES.

CREATE THE PERPENDICULAR BISECTOR OF THE BASE.

PRESS MENU, AND UNDER "CONSTRUCTION"

SELECT PERPENDICULAR BISECTOR.

THE LINE THAT'S DRAWN DIVIDES THE BASE IN TWO

AND IS PERPENDICULAR TO THE BASE.

THE TRIANGLE YOU WILL CONSTRUCT WILL HAVE A

VERTEX ALONG THE PERPENDICULAR BISECTOR.

PRESS MENU, AND UNDER "POINTS AND LINES"

SELECT SEGMENT.

CONSTRUCT A LINE SEGMENT FROM ONE OF THE ENDPOINTS

OF THE BASE TO THE PERPENDICULAR BISECTOR.

THEN CONSTRUCT A SECOND LINE FROM THE POINT

ON THE BISECTOR TO THE OTHER ENDPOINT OF THE BASE.

YOU SHOULD NOW HAVE A TRIANGLE CONSTRUCTED

SIMILAR TO THE ONE SHOWN HERE.

MEASURE EACH OF THE SIDES OF THE TRIANGLE.

PRESS MENU, AND UNDER "MEASUREMENT" SELECT LENGTH.

USE THE NAV PAD TO MOVE THE POINTER OVER EACH SIDE.

YOU WILL SEE THE MEASUREMENT APPEAR.

PRESS ENTER ONCE TO SELECT THE MEASUREMENT.

MOVE THE POINTER SO THAT THE MEASUREMENT APPEARS

NEXT TO THE SIDE AND PRESS ENTER AGAIN.

REPEAT THIS PROCESS FOR EACH SIDE OF THE TRIANGLE

AND THE RECTANGLE.

TRY TO GET YOUR SCREEN TO LOOK LIKE THIS.

WHAT DO YOU NOTICE ABOUT THIS TRIANGLE?

DO YOU SEE THE TWO SIDES ARE EQUAL?

IF YOU MOVE THE POINT ALONG THE PERPENDICULAR BISECTOR

THE SIDE LENGTHS CHANGE.

BUT IN EACH CASE THEY ARE EQUAL TO EACH OTHER.

THIS TYPE OF TRIANGLE IS CALLED

AN ISOSCELES TRIANGLE.

WITH AN ISOSCELES TRIANGLE TWO SIDES HAVE

EQUAL LENGTH AND ARE CONGRUENT TO EACH OTHER.

PRESS THE ESCAPE BUTTON TO EXIT MEASUREMENT MODE.

NOW THAT YOU HAVE THE MEASUREMENTS DISPLAYED

YOU CAN SET THE VALUES TO SPECIFIC AMOUNTS

FOR THE BASE.

USE THE NAV PAD TO MOVE THE POINTER ABOVE THE BASE.

PRESS ENTER AND YOU WILL BE ABLE TO EDIT

THE TEXT OF THE MEASUREMENT.

CHANGE THE LENGTH TO 5.

CHANGE THE SIDE LENGTHS TO MATCH THE TRIANGLE SHOWN.

TO DO SO, HOVER OVER THE TOP POINT OF THE TRIANGLE,

SELECT IT AND SLIDE IT UP AND DOWN.

CHANGE THE LENGTHS TO 6 FOR EACH SIDE.

OR, DEPENDING ON YOUR DISPLAY,

AS CLOSE TO 6 AS POSSIBLE.

TRY TO GET YOUR SCREEN TO LOOK LIKE THIS.

NOW HERE IS WHERE A TRIANGLE PROVES ITS STABILITY.

LOOKING AT THE TRIANGLE YOU HAVE CONSTRUCTED,

IS IT POSSIBLE TO MOVE ANY OF THE POINTS ON THE TRIANGLE

WHILE STILL KEEPING THE SIDE LENGTHS 6+6+5 AS SHOWN?

TRY DIFFERENT CONFIGURATIONS OF THE TRIANGLE

AND YOU WILL SEE THAT IT IS IMPOSSIBLE TO DO.

FOR EXAMPLE, IF YOU MOVE THE TOP POINT UP OR DOWN,

BOTH SIDE LENGTHS CHANGE.

YOU CAN KEEP ONE OF THE SIDE LENGTHS THE SAME

BUT THE OTHER SIDE LENGTH CHANGES.

ALSO, CHANGING THE LENGTH OF THE BASE

CHANGES ONE OR BOTH SIDE LENGTHS.

FINALLY, MOVING ANY TWO OR ALL THREE OF THE POINTS

CHANGES THE TRIANGLE.

THE DIFFICULTY OF CREATING ANOTHER TRIANGLE

WITH THE SAME SIDE LENGTHS IS IN STARK CONTRAST

TO THE EASE OF DOING THAT WITH A RECTANGLE.

TAKE A LOOK AT THE FOLLOWING RECTANGLE.

IS IT POSSIBLE TO CONSTRUCT ANOTHER FOUR SIDED FIGURE

WITH THE SAME SIDE LENGTHS?

IN FACT, IT'S QUITE EASY.

CHANGING THE ANGLES BETWEEN THE BASE AND SIDES

ALLOWS YOU TO CREATE AN INFINITE NUMBER OF

QUADRILATERALS WITH THESE SAME MEASUREMENTS.

NOW IMAGINE RATHER THAN GEOMETRIC SHAPES

YOU HAVE BUILDING MATERIALS IN THESE SHAPES.

THE TRIANGULAR SHAPED MATERIAL HAS A

BUILT-IN GEOMETRY THAT MAKES IT INFLEXIBLE

WHILE THE QUADRILATERAL SHAPE CAN BE RESHAPED

WHILE MAINTAINING THE LENGTH OF ITS SIDES.

AND THIS IS WHY TRIANGLES ARE USED

SO OFTEN IN BRIDGES, BUILDINGS,

AND LANDMARKS LIKE THE EIFFEL TOWER.

IN FACT, THIS PROPERTY OF TRIANGLES IS SO IMPORTANT

THAT IT GIVES RISE TO AN IMPORTANT

TRIANGLE POSTULATE:

LOOK AT TRIANGLES A, B, C AND D, E, F.

SUPPOSE THAT SIDE AB IS CONGRUENT TO DE.

SIDE BC IS CONGRUENT TO EF, AND AC IS CONGRUENT TO DF.

THEN WE CAN CONCLUDE THAT BOTH TRIANGLES ARE CONGRUENT

AND THIS MEANS THAT ANGLE A IS CONGRUENT TO ANGLE D;

ANGLE B IS CONGRUENT TO ANGLE E;

AND ANGLE C IS CONGRUENT TO ANGLE F.

THIS IS KNOWN AS THE SIDE-SIDE-SIDE POSTULATE

AND IS A POWERFUL GEOMETRIC TOOL.

LET'S PUT THIS IDEA TO IMMEDIATE USE WITH OUR

ANALYSIS OF THE EIFFEL TOWER STRUCTURE.

FIRST LET'S RETURN TO THE BASIC STRUCTURE

OF A TRUSS THAT WE'VE BEEN ANALYZING.

IT SHOULD COME AS NO SURPRISE TO YOU THAT THE

TYPICAL TRIANGULAR TRUSS IS AN ISOSCELES TRIANGLE.

WHAT ARE THE ADVANTAGES TO THIS TYPE OF TRIANGLE?

SINCE THE TRUSS REDIRECTS AND REDISTRIBUTES

THE WEIGHT OF THE LOAD, THEN AN ISOSCELES TRUSS

WILL REDISTRIBUTE THE FORCE EVENLY.

AS AN ENGINEER YOU WOULD NOT WANT TO OVERLOAD

ONE PART OF THE BRIDGE AT THE EXPENSE OF ANOTHER PART.

SO THE EIFFEL TOWER HAS MANY TRUSSES

THROUGHOUT THE STRUCTURE.

BUT ONE COMMON TYPE OF TRUSS

FOUND IN THE STRUCTURE IS SHOWN HERE.

NOTICE THAT IT HAS AN ADDITIONAL VERTICAL SUPPORT.

BASICALLY, THE ONE ISOSCELES TRIANGLE

IS SPLIT INTO TWO TRIANGLES.

BUT WHAT IS HAPPENING GEOMETRICALLY?

RECALL FROM YOUR EXPLORATION THAT FOR AN

ISOSCELES TRIANGLE THE TOP VERTEX IS ON THE SAME LINE

AS THE PERPENDICULAR BISECTOR OF THE BASE.

THIS MEANS THAT THE BASE OF THE ORIGINAL TRIANGLE

IS CUT IN HALF.

LET'S NOW LOOK AT THE TWO SMALLER TRIANGLES.

WE KNOW THAT THESE TWO SIDES ARE CONGRUENT SINCE THEY ARE

THE SAME SIDES FROM THE ORIGINAL ISOSCELES TRIANGLE.

BECAUSE OF THE PERPENDICULAR BISECTOR,

THESE TWO SIDES OF THE TWO TRIANGLES ARE CONGRUENT.

FINALLY, SINCE THE TWO SMALLER TRIANGLES

SHARE THE SAME SIDE,

IT IS BY DEFINITION CONGRUENT TO ITSELF.

AS A RESULT OF THE SIDE-SIDE-SIDE POSTULATE

WE CAN CONCLUDE THAT THE TWO SMALLER TRIANGLES

ARE CONGRUENT TO EACH OTHER.

THIS MEANS THAT THESE TWO ANGLES ARE CONGRUENT,

AND THESE TWO ANGLES ARE CONGRUENT.

NOTICE THAT SINCE THESE ANGLES

ARE ALSO PART OF THE ISOSCELES TRIANGLE,

THEN WE CAN CONCLUDE THAT

FOR ANY ISOSCELES TRIANGLE

THE BASE ANGLES ARE CONGRUENT.

SO THE TWO TRIANGLES FORMED FROM THE

VERTICAL SUPPORT AND THE TRUSS ARE CONGRUENT.

BUT THERE'S MORE.

RECALL THAT THE PERPENDICULAR BISECTOR,

BY DEFINITION, CREATES TWO 90 DEGREE ANGLES.

THIS MEANS THAT THE TWO CONGRUENT TRIANGLES

ARE RIGHT TRIANGLES.

THE PERPENDICULAR BISECTOR SPLITS THE BASE

OF THE ISOSCELES TRIANGLE INTO TWO EQUAL PARTS.

THE SAME HAPPENS TO THE ANGLE OF THE TOP VERTEX.

HOW DO WE KNOW THIS?

SINCE THE TWO RIGHT TRIANGLES ARE CONGRUENT,

THEN IT FOLLOWS THAT THESE TWO ANGLES ARE CONGRUENT.

THESE TWO ANGLES MAKE UP A SINGLE ANGLE

OF THE ISOSCELES TRIANGLE.

SO THE PERPENDICULAR BISECTOR IS ALSO

THE ANGLE BISECTOR OF THE ISOSCELES TRIANGLE.

NOW YOU KNOW HOW TO BUILD A TRUSS.

FIRST, TAKE TWO EQUAL PIECES THAT WILL MAKE UP

THE CONGRUENT SIDES OF THE ISOSCELES TRIANGLE.

TAKE A THIRD SIDE TO SERVE AS THE BASE OF THE TRIANGLE.

YOU ARE LIMITED IN TERMS OF THE SIZE OF THE BASE.

IF YOU EXTEND THE TWO SIDES AS FAR AS YOU CAN

SO THAT THE ANGLE BETWEEN THE TWO SIDES IS 180 DEGREES,

THEN THE BASE OF THE TRIANGLE

CANNOT BE ANY LONGER THAN THIS.

IN FACT, THIS BRINGS UP AN INEQUALITY

TRUE OF ALL TRIANGLES.

THE SUM OF THE THIRD SIDE OF A TRIANGLE

IS LESS THAN THE SUM OF THE OTHER TWO SIDES.

SO OUR TRUSS HAS TWO EQUAL SIDES

AND A BASE TO FORM AN ISOSCELES TRIANGLE.

IN ORDER TO ADD THE VERTICAL SUPPORT, SIMPLY MEASURE THE

ANGLE OF THE TOP VERTEX AND FIND THE ANGLE BISECTOR.

THIS ANGLE BISECTOR IS ALSO THE

PERPENDICULAR BISECTOR OF THE BASE

AT THE POINT WHERE THE BISECTOR INTERSECTS THE BASE.

A TRUSS INHERITS THE STRONG GEOMETRY OF A TRIANGLE.

THIS INFLEXIBILITY ALLOWS FOR STURDY STRUCTURES

LIKE THE EIFFEL TOWER.

NOW YOU SAW THAT WITH A BRIDGE THE TRUSSES ARE

ARRANGED HORIZONTALLY, BUT NOTICE THAT THE EIFFEL TOWER

HAS MANY TRUSSES THAT ARE ALIGNED VERTICALLY.

WHAT COULD BE THE REASON FOR THIS?

SINCE THE PURPOSE OF TRIANGULAR TRUSSES

IS FOR SUPPORT, THEN A SET OF VERTICAL TRUSSES,

LIKE THE ONES ON THE EIFFEL TOWER,

WOULD BE TO DEAL WITH A SIDEWAYS FORCE.

IN THE CASE OF THE EIFFEL TOWER

THE SIDEWAYS FORCES ARE FROM THE WIND.

TALL STRUCTURES NEED THE STRENGTH OF TRUSSES

TO WITHSTAND THE OFTEN STRONG FORCES OF WIND.

BUT THE WIND COMES FROM DIFFERENT DIRECTIONS.

A TRUSS NEEDS TO ALLOW FOR THIS.

FOR EXAMPLE, WHEN THE WIND COMES FROM THIS DIRECTION,

THE FORCES OF COMPRESSION ARE HERE

AND THE FORCE OF TENSION IS HERE.

WHEN THE WIND COMES FROM THIS DIRECTION

THE FORCES OF COMPRESSION ARE HERE

AND THE FORCE OF TENSION IS HERE.

FINALLY, WHEN THE WIND COMES FROM THIS DIRECTION

THE COMPRESSION FORCES ARE HERE

AND THE TENSION FORCES HERE.

BECAUSE THE TRUSSES ON THE EIFFEL TOWER

ARE LITERALLY SURROUNDED BY WIND,

THEN ONE POSSIBLE SOLUTION IS FOR THE

TRIANGULAR STRUCTURE TO HAVE EQUAL SIDES.

IN OTHER WORDS, TRIANGULAR TRUSSES

THAT ARE EQUILATERAL TRIANGLES.

WITH AN EQUILATERAL TRIANGLE

ALL THREE SIDES ARE CONGRUENT.

THIS TYPE OF TRIANGLE IS STILL AN ISOSCELES TRIANGLE,

BUT RATHER THAN TWO SIDES BEING CONGRUENT,

ALL THREE SIDES ARE.

THIS HAS IMPLICATIONS FOR THE ANGLE MEASUREMENTS

OF AN EQUILATERAL TRIANGLE.

LOOK AT EQUILATERAL TRIANGLE A, B, C.

BY DEFINITION, ALL SIDE LENGTHS ARE CONGRUENT

SO ANY TWO SIDES ARE CONGRUENT.

SEEN THIS WAY, AS AN ISOSCELES TRIANGLE,

WE KNOW THAT ANGLE A AND ANGLE B ARE CONGRUENT.

BUT SEEN THIS WAY WE KNOW THAT ANGLE B AND ANGLE C

ARE CONGRUENT.

IF ANGLE A IS CONGRUENT TO ANGLE B

AND ANGLE B IS CONGRUENT TO ANGLE C

THEN ALL THREE ANGLES ARE CONGRUENT TO EACH OTHER.

SINCE THE SUM OF THE ANGLES OF A TRIANGLE

IS 180 DEGREES,

WE DERIVE THE EQUATION 3X EQUALS 180 DEGREES

WHERE X IS THE MEASURE OF ANGLES A, B AND C.

SOLVING FOR X WE FIND THAT THE ANGLE MEASURES

OF AN EQUILATERAL TRIANGLE ARE 60 DEGREES.

MANY BUILDINGS HAVE TRIANGULAR TRUSSES

THAT ARE MADE UP OF EQUILATERAL TRIANGLES.

BUT ANOTHER OPTION IS TWO SETS OF ISOSCELES

TRIANGLE TRUSSES IN THE SHAPE OF A QUADRILATERAL.

NOTICE HOW THIS STYLE OF TRUSSES

IS USED THROUGHOUT THE EIFFEL TOWER.

BUT ONE OTHER THING TO NOTICE IS THAT

IN GOING FROM THE GROUND TO THE TOP OF THE TOWER

THE SETS OF TRUSSES GET SMALLER

YET THEY SEEM TO HAVE THE SAME SHAPE.

WHAT IS HAPPENING GEOMETRICALLY?

WHEN TWO TRIANGLES HAVE THE SAME SHAPE

THEY ARE SAID TO BE SIMILAR.

WHEN TWO TRIANGLES ARE SIMILAR

THEY HAVE CONGRUENT ANGLES BUT NOT CONGRUENT SIDES.

THE SIDES ARE PROPORTIONAL TO EACH OTHER.

FOR EXAMPLE, THESE TWO TRIANGLES ARE SIMILAR.

THIS MEANS THAT ANGLE A IS CONGRUENT TO ANGLE D.

ANGLE B IS CONGRUENT TO ANGLE E,

AND ANGLE C IS CONGRUENT TO ANGLE F.

ALTHOUGH THE CORRESPONDING SIDES OF THE TRIANGLE

ARE NOT CONGRUENT, THEY ARE PROPORTIONAL.

SO IF SIDE AB IS THREE TIMES LONGER THAN SIDE DE,

THEN THE OTHER SIDES OF TRIANGLE ABC

ARE THREE TIMES LONGER THAN

THE CORRESPONDING SIDES OF TRIANGLE DEF.

SO HOW DO WE KNOW THAT THE TRIANGULAR TRUSSES

OF THE EIFFEL TOWER FORM SIMILAR TRIANGLES?

WE HAVE TO MAKE SOME ASSUMPTIONS AND

DRAW CONCLUSIONS BASED ON THOSE ASSUMPTIONS.

FIRST, LET'S LOOK AT THE FIRST SET OF TRUSSES,

THE LARGER ONES.

WE KNOW THAT EACH SIDE OF THIS FOUR-SIDED FIGURE

IS MADE UP OF AN ISOSCELES TRIANGLE

AND THE VERTICAL SUPPORT IS THE PERPENDICULAR BISECTOR

OF THE BASE OF THE TRIANGLE.

LET'S ASSUME THAT BOTH OF THESE ISOSCELES TRIANGLES

ARE CONGRUENT.

SECOND, LET'S ASSUME THAT ALL THE TRIANGLES IN THE

NETWORK OF TRUSSES ARE ISOSCELES TRIANGLES.

NEXT, LET'S ASSUME THAT THE VERTICES

OF THIS NETWORK OF TRUSSES ARE COLLINEAR.

FINALLY, WE ASSUME THAT THE TRUSSES

INTERSECT AT THESE VERTICES.

FROM AN ARCHITECTURAL POINT OF VIEW

THESE ASSUMPTIONS ARE REASONABLE

AND HELP TO MAKE THE TOWER STURDY.

BUT FROM A GEOMETRIC PERSPECTIVE

THESE ASSUMPTIONS NEED TO BE EXPLICITLY STATED.

MATH IS ABOUT PROOF BASED ON A SET OF GIVEN

DEFINITIONS AND ASSUMPTIONS.

WE WANT TO PROVE THAT EACH SET OF ISOSCELES TRIANGLES

IS SIMILAR TO THE OTHERS.

IN FACT, ALL WE NEED TO DO IS SHOW THAT

THESE TWO TRIANGLES ARE SIMILAR.

ONCE WE DO THAT, THEN WE CAN EXTEND THE RESULTS

TO THE OTHER TRIANGLES.

SO LET'S START THERE.

WE JUST NEED TO SHOW THAT THE CORRESPONDING ANGLES

OF EACH TRIANGLE ARE CONGRUENT.

THESE TWO ANGLES ARE VERTICAL ANGLES

BECAUSE BY DEFINITION ALL VERTICAL ANGLES

ARE CONGRUENT TO EACH OTHER.

SINCE WE STATED THAT ALL OF THE TRIANGULAR SUPPORTS

ARE ISOSCELES TRIANGLES, THEN IT FOLLOWS

THAT THESE TWO ANGLES ARE CONGRUENT.

FURTHERMORE, SINCE THIS TRIANGLE IS ISOSCELES

THEN WE KNOW THAT THIS PART OF THE TRIANGLE IS THE

PERPENDICULAR BISECTOR OF THE BASE OF THE TRIANGLE.

NOTICE THERE ARE FOUR RIGHT ANGLES FORMED AT

EACH INTERSECTION OF THE PERPENDICULAR BISECTOR

AND THE TRIANGLE BASES.

USING THE PARALLEL POSTULATE WE KNOW THAT

BECAUSE THE INTERIOR ANGLES EQUAL 180 DEGREES

THEN THESE TWO LINES ARE PARALLEL.

SINCE THE BASES OF THE TWO TRIANGLES ARE PARALLEL,

THEN THIS LINE IS A TRANSVERSAL

THAT INTERSECTS THE TWO PARALLEL LINES.

ONCE AGAIN, USING THE PARALLEL POSTULATE WE KNOW

THAT ALTERNATE INTERIOR ANGLES ARE CONGRUENT.

IN OTHER WORDS, THE CORRESPONDING ANGLES

FOR EACH ISOSCELES TRIANGLE ARE CONGRUENT.

SINCE WE HAVE SHOWN THAT ALL THREE

CORRESPONDING ANGLES ARE CONGRUENT,

THEN THE TWO TRIANGLES ARE SIMILAR.

YOU CAN USE THE SAME ANALYSIS FOR THE

REMAINING SETS OF TRIANGLES TO SHOW THAT

ALL THE TRIANGULAR TRUSSES ARE SIMILAR TO EACH OTHER.

THE EIFFEL TOWER IS A STUDY IN TRIANGULAR FORMS.

BUILT BY SOMEONE MORE ACCUSTOMED TO

BUILDING BRIDGES, THE TOWER ITSELF IS A KIND OF BRIDGE.

BUILT AT THE END OF THE 19TH CENTURY

IT SERVED AS A BRIDGE TO THE 20TH CENTURY

WITH ITS DAZZLING TECHNOLOGICAL INNOVATIONS.

AND IT GREETS THE 21ST CENTURY

NO LONGER A SYMBOL OF MODERNISM

BUT A CLASSIC FORM THAT DEFINES

ONE OF THE GREAT CITIES OF THE WORLD.